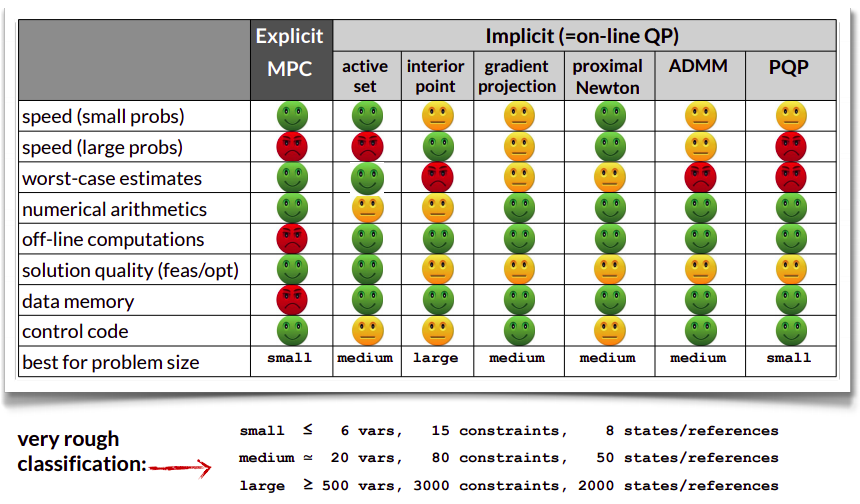
**Matlab solvers**

**fmincon**

Most used algorithm for QP are:

* active set
* interior-point
* gradient projection
* alternating direction method of multipliers



**Robust MPC**

**Tube-Based RMPC**

In presence of bounded uncertainties, all the possible trajectories combinations of an uncertainty system lie in a bounded space of a nominal trajectory, called tube.

Tubes allows to satisfy of the constraints for all the disturbance sequence ensuring that the nominal trajectory satisfies suitably tightened constraints.

**Dynamic system**

Considering the uncertain linear system:

where: solution of the uncertain system

, state disturbance, state disturbance constraint set

Considering a nominal linear system:

where: is solution of the nominal system

**Preamble**

**Open-loop tube**

Considering an uncertainty linear system and a nominal one, the deviation of the actual state from the nominal state is:

The error evolution derived, for , is:

The uncertainty set is defined accordingly:

Finally, the tube is defined as:

where:

nominal state defining the centre of the tube

It can be proof that with an open loop control, the tube increases in dimension as i increases, since the uncertainty set increases and then the trajectory can diverge.

The solution to this is using a feedback control to control the size of and then make the actual trajectory converge to the nominal one.

**Closed-loop tube**

The feedback control law proposed is:

The state x satisfies the difference equation:

The deviation and the error evolution:

The error evolution derived, for , is:

The uncertainty set is defined accordingly:

The tube generated is:

where:

nominal state defining the centre of the tube

**Existence of the minimal robust positive invariant set**

The uncertainty set at a certain instant can be replaced by the:

* Minimal robust positive invariant set to obtain a more conservative outer-bounding tube
* to obtain a less conservative outer-bounding tube

**Origin condition relaxation**

Furthermore since exists whenever is stable, can be proven that it is possible to relax the condition .

**Constraints**

Defining the uncertain and nominal system constraint set as:

The tube-based MPC is made up by:

1. Nominal state-control trajectory that satisfies the tightened constraint:
2. Feedback controller that track the nominal trajectory

where: deviation of the actual state

The nominal state-control trajectory can be:

* generated once at initial time
* generated sequentially using standard MPC for deterministic system

**Nominal trajectory**

For the nominal system

it is solved the standard optimal control problem:

which solution gives the control law:

Then the controlled nominal system is described by:

**Feedback controller**

The feedback controller generates the (proposed) control law:

Then the controlled system is described by:

where: deviation of the actual state, satisfies:

which is the error evolution.

**System state and dynamic model**

Considering two systems:

1. system with state satisfies:
2. system with state satisfies:

Since they are proportional with through an invertible matrix T, they are equivalent.

**Uncertainty set S**

Setting the initial condition:

**Constraints**

**Tightened constraint**

Suppose is a polytope and described by a set of scalar inequalities:

It can be proof that constraints of this form can be replaced by constraint of the form:

ensuring that the satisfaction of this the tightened constraint by the nominal system ensures that the uncertain system satisfies the original constraint.

**Time-varying Tightened constraint**

The constant tightened constraint set is conservative, reducing unnecessarily the feasible set . This conservativeness can be decreased using time-varying constraint set:

**Final Notes**

* Computation of tightened constraints may be expensive